Research Summary

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1 Phase locked loops

1. Phase locked Loops, J Encas, 1993. Published by Chapman and Hall. ISBN 0-412-48260-6.

- A phase-locked loop (PLL) is a circuit which synchronises the frequency of the output of an oscillator with the frequency of a reference signal.
- The phase difference of the two signals is used to perform this synchronisation.
- This could be useful in our case using one diode as the reference and the other as the oscillator to persuade them to oscillate at the same frequency and in phase.
- The system consists of three main blocks
  - A phase comparator (PC), which outputs a voltage proportional to the phase difference between the reference signal and the oscillator.
  - A low pass filter
  - A voltage controlled oscillator
- When operating in the locked state, the frequency of the reference signal and the VCO are the same.
- Using a systems such as this may be possible if the bias voltage on the Gunn diodes affects their frequency of oscillation.
- Phase comparators that operate at our target frequency may be difficult to obtain.

2 Injection locking

1. RF power combining and injection locking of oscillators, RJ Collier and AM Affandi, IEEE Colloquium on RF Combining, 12 April 1990.

2. A study of locking phenomena in oscillators, R Adler, Proceedings of the Institute of Radio Engineers, Waves and Electrons, June 1946, pp351–357.

- Injection locking appears to be a more passive way of synchronising the frequency of oscillation of multiple oscillators for power combining.
- The range of frequencies at which locking can occur is given by:

$$\frac{f_0}{2Q_{\text{ext}}}\sqrt{\frac{P_i}{P_0}}\sin\alpha$$

Where:

f_0	Frequency
Q_{ext}	Q value of the reference oscillator
P_i	Injected power from the reference oscillator
P_0	Power output from the controller oscillator
α	Phase angle between the signals P_i and P_0.

- As the frequency of the oscillator is shifted from its natural frequency by means of the injection locking, the phase difference, α, will increase.
- The greatest efficiency is obtained when α is at zero, dropping to 50% efficiency as $\alpha \to 90^\circ$.
- The more power than is injected (as a proportion of the output power), the smaller the phase difference in the oscillations.
- Spectral purity of injection locked combined oscillators is often better than for single oscillators.
- Explanation of locking range:
- Variables used:

ω_0	Controlled oscillator frequency.

ω_1	Impressed frequency.
ω	Instantaneous frequency.
$\Delta\omega_0$	$\omega_0 - \omega_1$.
$\Delta\omega$	$\omega - \omega_1$.
T, T'	Time constant, decay time.
E, E_1	Controlled and impressed signals.
ϕ	Phase difference.
A	$\frac{d\phi}{d\omega}$.

- $T \ll \frac{1}{\Delta\omega_0}$, $T' \ll \frac{1}{\Delta\omega_0}$
- If a signal of frequency $\omega_1 = \omega_0 + \Delta\omega_0$ is added to the signal E, a beat frequency will result. The frequency will be $\ll \omega_0$.
- Phase difference:

$$\begin{aligned} \phi &= A(\omega - \omega_0) \\ &= A\left[(\omega - \omega_1) - (\omega_0 - \omega_1)\right] \\ &= A[\Delta\omega - \Delta\omega_0] \end{aligned}$$

- Signal difference:

$$-\frac{E_1}{E}\sin\alpha = A\left[\frac{d\alpha}{dt} - \Delta\omega_0\right]$$

- Let $B = \frac{E_1}{EA}$:

$$\begin{aligned} \frac{d\alpha}{dt} &= -B\sin\alpha + \Delta\omega_0 \\ \tan\phi &= 2Q\frac{\Delta\omega}{\omega_0} \end{aligned}$$

- For small ϕ:

$$\begin{aligned} \phi &\approx 2Q\frac{\Delta\omega}{\omega_0} \\ A &= \frac{2Q}{\omega_0} \\ B &= \frac{E_1}{E}\frac{\omega_0}{2Q} \\ \frac{d\alpha}{dt} &= -\frac{E_1}{E}\frac{\omega_0}{2Q}\sin\alpha + \Delta\omega_0 \end{aligned}$$

- For steady state:

$$\begin{aligned} 0 &= -\frac{E_1}{E}\frac{\omega_0}{2Q}\sin\alpha + \Delta\omega_0 \\ \sin\alpha &= 2Q\frac{E}{E_1}\frac{\Delta\omega_0}{\omega_0} \end{aligned}$$

- This gives α, the stationary phase angle between the two signals as $\alpha = \sin^{-1}(2Q\frac{E}{E_1}\frac{\Delta\omega_0}{\omega_0})$.

- Since $-1 \leq \sin\alpha \leq 1$ for all α:

$$\begin{aligned} \left|2Q\frac{E}{E_1}\frac{\Delta\omega_0}{\omega_0}\right| &< 1 \\ \frac{E_1}{E} &> 2Q\left|\frac{\Delta\omega_0}{\omega_0}\right| \end{aligned}$$